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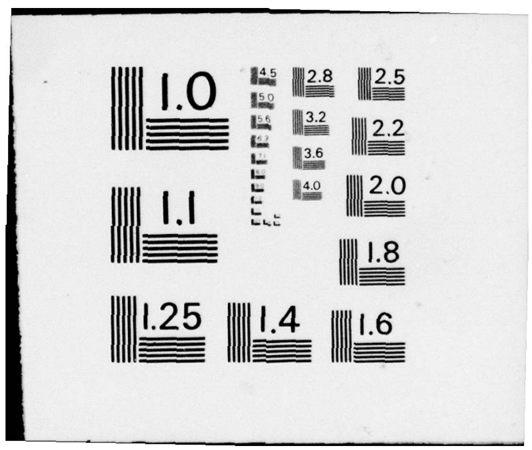
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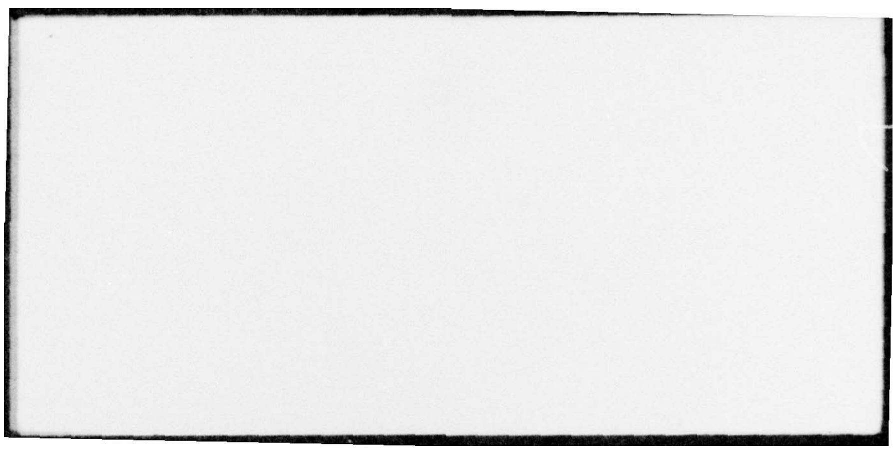

NETWORK MODELS FOR PRODUCTION SCHEDULING PROBLEMS WITH CONVEX COST AND BATCH PROCESSING

Research Report 76-18

by

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August, 1976

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This paper considers a class of production scheduling problems which can be modeled as network flow problems. The problems are addressed under the assumption that production occurs in batches. We also require that the cost function be convex and separable. The model applies for a number of well known production scheduling problems and will handle multiple products and multiple facilities.

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TABLE OF CONTENTS

Abstract

This paper considers a class of production scheduling problems which can be modeled as network flow problems. The problems are addressed under the assumption that production occurs in batches. We also require that the cost function be convex and separable. The model applies for a number of well known production scheduling problems and will handle multiple products and multiple facilities.

This paper considers the problem of producing N products on M identical facilities where each product i is produced in batches of size s_i. We assume that the planning horizon is made up of H equal length production periods where each facility is capable of producing one batch of any product during a production period. We also assume that the total cost can be expressed as

$$\text{(I)} \qquad \sum_{i=1}^{N} \sum_{t=1}^{H} a_{it}(x_{it}) + \sum_{i=1}^{N} \sum_{t=1}^{H} q_{it}\left(\sum_{j=1}^{t-1} x_{ij}\right)$$

where $a_{it}(\cdot)$ is a convex function of x_{it}, the number of batches of product i produced in period t, and $q_{it}(\cdot)$ is a convex function of $\sum_{i=1}^{t-1} x_{it}$, the total number of batches produced before period t. (All that is actually required is that the piecewise linear approximations over $x_{it} = 0, 1, 2, \ldots$ and $\sum_{j=1}^{t-1} x_{it} = 0, 1, 2, \ldots$ be convex for all $i = 1, 2, \ldots, N$ and all $t = 1, 2, \ldots, H$.) Under these assumptions, we show that an optimum production schedule can be efficiently generated using any of the very efficient minimum cost flow algorithms.

Special cases of this problem were posed as minimum cost flow problems by Dorsey, Hodgson, and Ratliff [2] [3]. Their procedure involved posing the problem as an integer program and then transforming it to a minimum cost flow problem. The procedure given here is somewhat more general and hopefully more easily understood. Other special cases were considered by Elmaghraby and Mallik [5], Elmaghraby, Mallik, and Nuttle [6], and Elmaghraby and Bawle [4]. Bowman [1] posed a special case of the one product problem as a transportation problem. Related problems under the same convex cost and separability assumptions are reviewed by Veinott [7].

The Network Representation

First consider the case where there is only a single product i. This is represented by the network in Figure 1 for a three period problem. If the cost functions are of the form (I), then the problem of defining a set of x_{it} to minimize (I) can obviously be solved by finding a minimum cost flow from S to T. Since all arc costs are convex, any of the standard minimum cost flow algorithms will solve the problem (Hu [7]). In fact for some cases (e.g., Dorsey, Hodgson, Ratliff [2]) they can be considerably improved by taking advantage of the network structure. Note that upper and lower bounds on the x_{it} or the $\sum_{j=1}^{t-1} x_{ij}$ for any t = 1, 2, ..., H can be handled by simply putting upper and lower bounds on the corresponding arc flows. This network model is a generalization of the model given by Fulkerson [6].

The model can be extended to include any number of products by expanding the network as illustrated in Figure 2. Using this model, upper and lower bounds on the total number of batches produced in a given period t (i.e., the number of facilities available in t) can be handled by putting upper and lower bounds on the arc corresponding to $\sum_{i=1}^{N} x_{it}$.

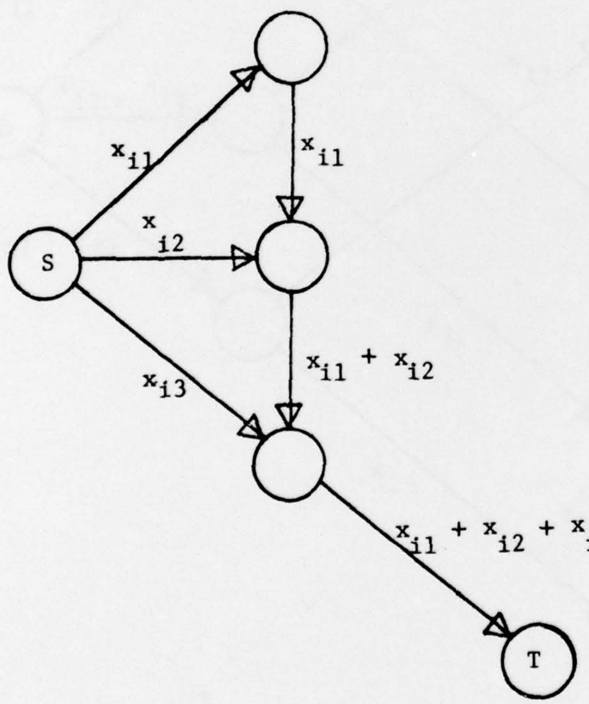

Figure 1: Network Corresponding to a Single Product Problem with Three Periods.

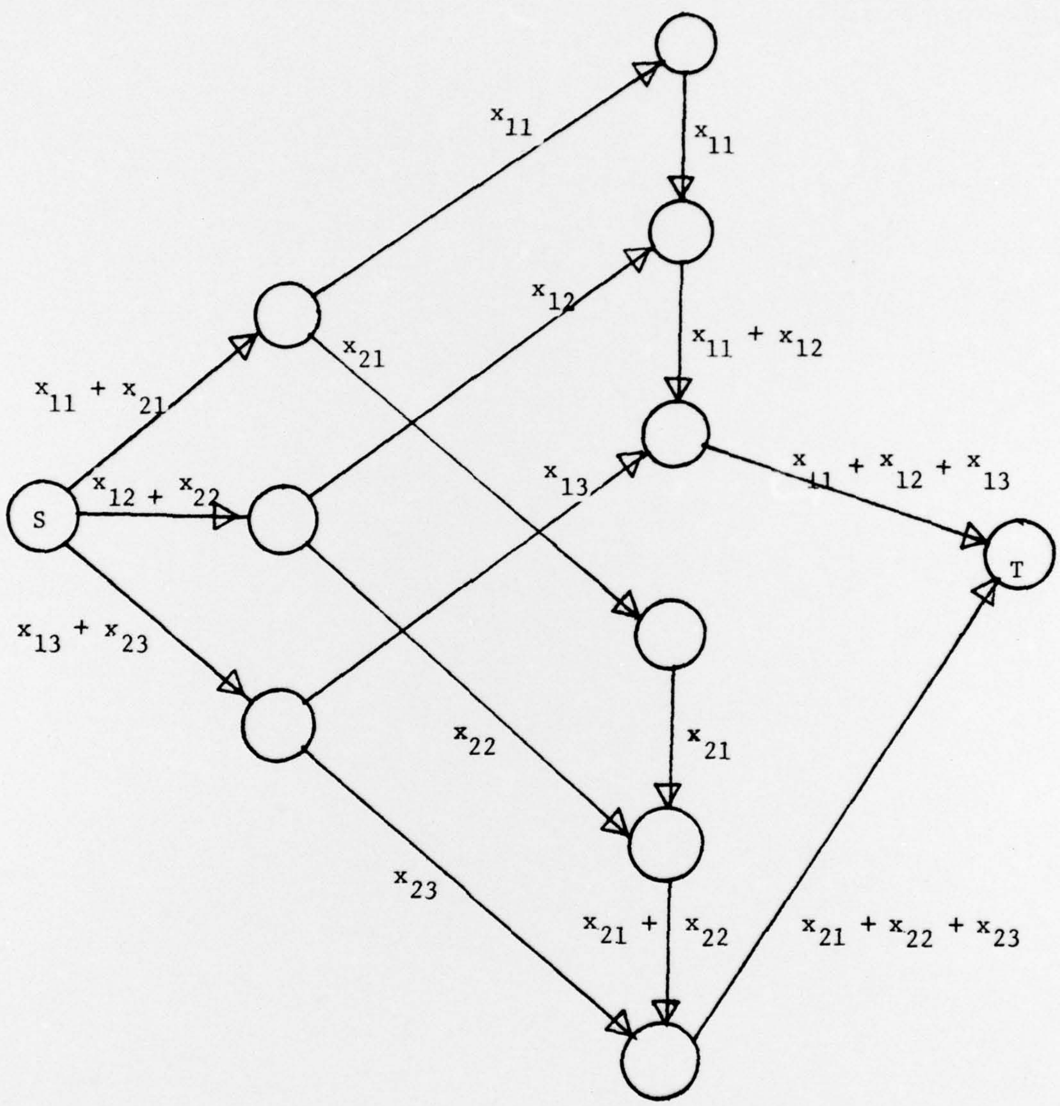

Figure 2: Network Corresponding to a Two Product Problem with Three Periods.

Modeling the Production Scheduling Problem

In spite of its simplicity, a surprising number of production scheduling problems can be solved using this network model. As an illustration consider a problem with known demands d_{it} for product i in period t and assume that the demands occur uniformly over the period t. Assume that the production of product i becomes available as a batch of size s_i at the end of the period in which it is produced. Assume that the production cost is p_{it} per batch for each batch of product i produced in period t. The inventory carrying cost is c_{it} per unit of product per unit time, and the backordering cost is b_{it} per unit of product per unit time. All costs are assumed to be nonnegative.

Let I_{it} denote the inventory level of product i at the beginning of period t. Since there is generally no reason in most systems for having at the same time both a positive amount of product i in inventory and a positive amount on backorder, backorders of product i at the beginning of period t will correspond to letting I_{it} be negative. The possible inventory fluctuation for a single period is illustrated in Figure 3.

It is easily shown that the inventory/backorder cost for product i during period t as a function of the inventory level at the beginning of the period is

$$\phi_{it}(I_{it}) = \begin{cases} b_{it}[(d_{it}/2) - I_{it}] & \text{for } I_{it} \leq 0 \\ c_{it}(I_{it}^2/2d_{it}) + b_{it}(d_{it} - I_{it})^2/2d_{it} & \text{for } 0 \leq I_{it} \leq d_{it} \\ c_{it}[I_{it} - (d_{it}/2)] & \text{for } I_{it} \geq d_{it} \end{cases}$$

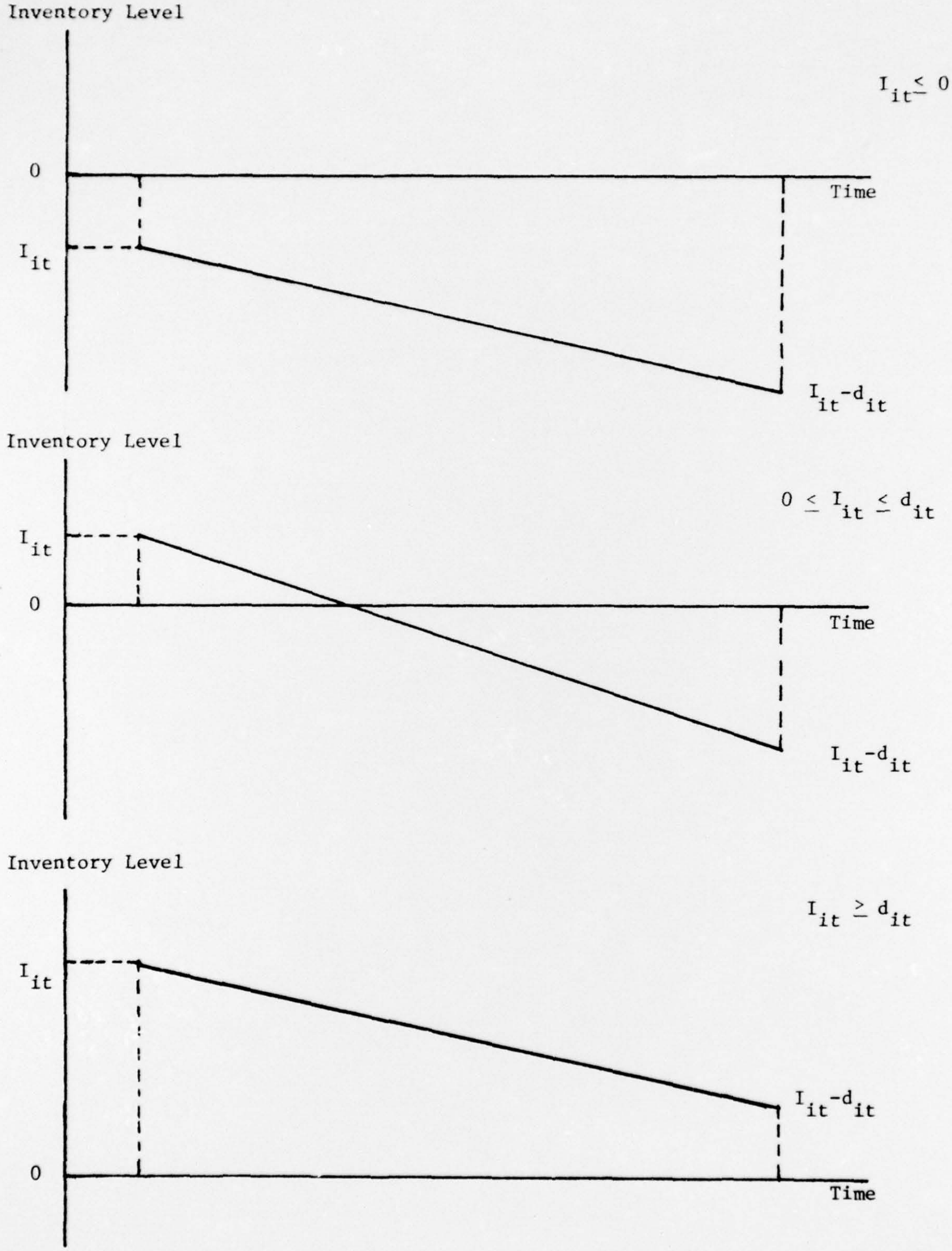

Figure 3: Possible Inventory Fluctuation

The function $\phi_{it}(I_{it})$ is sketched in Figure 4 and is convex.

Since the inventory level of product i at the beginning of period t can be expressed as $I_{it} = I_{i1} + s_i \sum_{j=1}^{t-1} x_{ij} - \sum_{j=1}^{t-1} d_{ij}$, we have that, $q_{it}(\sum_{j=1}^{t-1} x_{ij}) = \phi_{it}(I_{i1} + s_i \sum_{j=1}^{t-1} x_{ij} - \sum_{j=1}^{t-1} d_{ij})$ is convex in $\sum_{j=1}^{t-1} x_{ij}$. The piecewise linear approximation of $q_{it}(\sum_{j=1}^{t-1} x_{ij})$ on $\sum_{j=1}^{t-1} x_{ij} = 0, 1, 2, \ldots$ is illustrated by the dashed line in Figure 4 and is obviously convex. Since $a_{it}(x_{it}) = p_{it}x_{it}$ the cost functions satisfy all the assumptions for (I) and the network model is applicable.

Upper and lower bounds on the inventory level of any product i in period t can be incorporated into the model by putting the appropriate lower and upper bounds on the arc corresponding to $\sum_{j=1}^{t-1} x_{ij}$. Hence, the inventory levels of each product at the end of the planning horizon can either be fixed at specified values by setting the corresponding lower and upper bounds equal to each other, or allowed to fluctuate between specified bounds.

Under the same assumptions as above but with all demand assumed to occur at the end of a period the inventory/backorder cost becomes

$$\phi_{it}(I_{it}) = \begin{cases} -b_{it}I_{it} & \text{for } I_{it} \leq 0 \\ c_{it}I_{it} & \text{for } I_{it} \geq 0 \end{cases}$$

and the assumptions for (I) are again satisfied.

Under the same assumptions as above but with both production and demand assumed to occur at a constant rate and backorders not allowed the inventory cost in period t is

$$\begin{aligned} \phi_{it}(I_{it}, x_{it}) &= c_{it}I_{it} - c_{it}(d_{it} - s_i x_{it})/2 \\ &= c_{it}(\sum_{j=1}^{t-1} x_{ij} - \sum_{j=1}^{t-1} d_{ij}) - c_{it}(d_{it} - s_i x_{it})/2 \end{aligned}$$

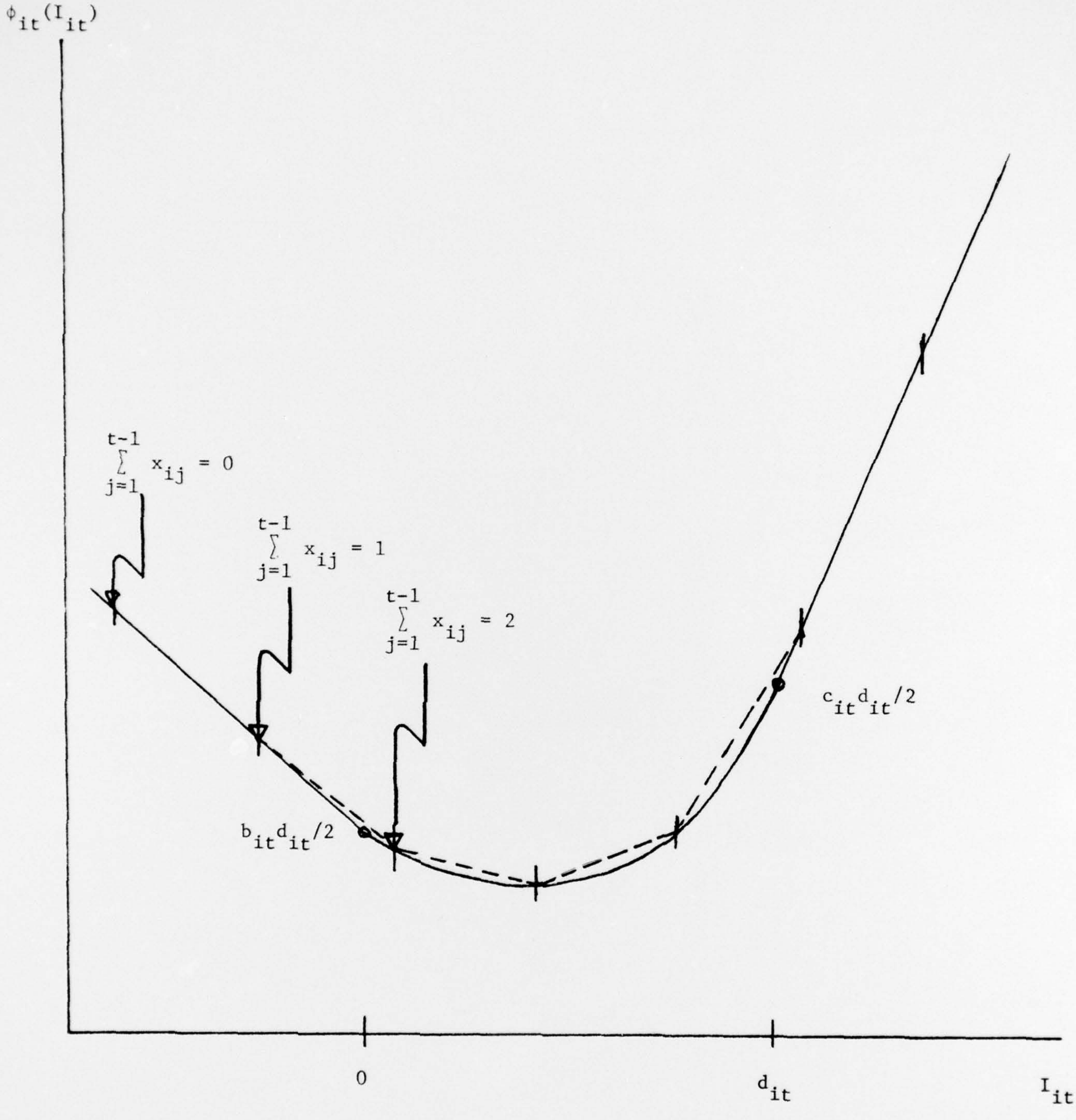

Figure 4: Cost Curve as a Function of Beginning Period Inventory

By letting

$$a_{it}(x_{it}) = p_{it}x_{it} - c_{it}(d_{it} - s_i x_{it})/2$$

and

$$q_{it}\left(\sum_{j=1}^{t-1} x_{ij}\right) = c_{it}\left(\sum_{j=1}^{t-1} x_{ij} - \sum_{j=1}^{t-1} d_{ij}\right)$$

we again have all the assumptions for (I) satisfied and the model applies.

If the inventory level for this latter problem is allowed to be negative (i.e., backorders allowed) as well as positive, the nonlinear cost component corresponding to the case where both positive and negative inventory levels occur in the same period t contains both x_{it} and I_{it}. Since there is no apparent way to separate this component into a function of x_{it} and a function of I_{it}, this problem appears appreciably more difficulty than the others considered here.

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